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3 (Sem-5/CBCS) PHY HC1

2024

PHYSICS

(Honours Core)

Paper : PHY-HC-5016

(Quantum Mechanics and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

(a) Eigenvalue of Hamiltonian operator is

(i) kinetic energy

(ii) potential energy

(iii) both (i) and (ii)

(iv) total energy

(b) Why $\psi = e^x$ is not an acceptable wave function in quantum mechanics ?

Contd.

(c) What do you mean by space quantisation of an atom ?

(d) The value of $\left[\hat{x}, \frac{\partial}{\partial x} \right]$ is

(i) 1

(ii) -1

(iii) $i\hbar$

(iv) $-i$

(e) What is the value of spin-orbit interaction energy for the ground state of hydrogen atom ?

(f) When does the probability density of a quantum mechanical oscillator approach that of a classical oscillator ?

(g) Can the Stern-Gerlach experiment be performed with ions instead of neutral atoms ?

2. Answer the following questions : $2 \times 4 = 8$

(a) Is the wave function $\psi(x) = e^{ikx}$ an eigenfunction of the kinetic energy operator T ? If yes, what is its eigenvalue ?

(b) What is a Gaussian wave packet ? Express its wave function.

(c) The one-dimensional wave function is given by $\psi(x) = \sqrt{a} e^{-ax}$. Find the probability of finding the particle between $x = \frac{1}{a}$ and $x = \frac{2}{a}$.

(d) Calculate the Lande's g factor for the ${}^2p_{3/2}$ state.

3. Answer **any three** of the following questions : $5 \times 3 = 15$

(a) State the conditions of "acceptability of wave function" in quantum mechanics with explanation.

(b) Obtain time-independent Schrödinger wave equation for a free particle in one dimension. Give a physical interpretation of the wave function $\psi(x, t)$. $4+1=5$

- (c) Find the expectation value of energy when the state of harmonic oscillator is described by the following wave function :

$$\psi(x,t) = \frac{1}{\sqrt{2}} [\psi_0(x,t) + \psi_1(x,t)]$$

where $\psi_0(x,t)$ and $\psi_1(x,t)$ are the wave functions for the ground state and first excited state respectively.

- (d) State Pauli's exclusion principle. An atomic state is denoted by 3p_2 . Determine the values of L , S and J and mention whether the above state is admissible or not. $2+3=5$
- (e) Discuss the significance of zero-point energy with reference to a linear harmonic oscillator. The energy of a linear harmonic oscillator in the third excited state is 0.1 eV. Find the frequency of vibration. $2+3=5$

4. Answer **any three** of the following questions: $10 \times 3 = 30$

- (a) (i) What is the need for normalization of a wave function? A wave function $\psi(x)$ is given by

$$\psi(x) = A_n \sin \frac{2n\pi x}{L} \text{ in the region}$$

$0 \leq x \leq L$. Find the value of A_n using normalization condition.

$1+4=5$

- (ii) Derive the continuity equation from the time-dependent Schrödinger equation of a particle moving in a real potential and give its physical significance. $4+1=5$

- (b) A particle of mass m is moving in a one-dimensional potential given by

$$V(x) = 0 \text{ for } 0 \leq x \leq L$$

$$V(x) = \infty \text{ for } x < 0 \text{ and } x > L$$

Using appropriate boundary conditions, solve the Schrödinger equation and find allowed energy values and normalized wave functions of the particle. Also plot the eigenfunctions corresponding to different eigenvalues. $8+2=10$

(c) Write the radial equation of hydrogen atom and solve it for obtaining its energy eigenvalues. $2+8=10$

(d) What is anomalous Zeeman effect? Discuss the quantum mechanical theory of anomalous Zeeman effect, with special reference to Zeeman pattern for D_1 and D_2 lines of sodium. $2+8=10$

(e) (i) Describe and explain LS and JJ couplings. Illustrate them with vector diagram. $2+2+4=8$

(ii) Determine the possible values of resultant angular momentum for two electrons having $j_1 = \frac{3}{2}$ and

$$j_2 = \frac{5}{2}. \quad 2$$

(f) (i) A beam of electrons enters a uniform magnetic field of flux density 1.2 Wb/m^2 in the z-direction. Find the energy difference between the electrons whose spins are parallel and anti-parallel to the field. 5

(ii) Write short note on **any one** of the following : 5

(i) Paschen-Back effect

(ii) Stark effect