Total number of printed pages–7

3 (Sem–6/CBCS) MAT HE 2

2025

MATHEMATICS

(Honours Elective)

Paper : MAT–HE–6026

(Biomathematics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times10=10$

   (a) The difference equation

   $x_{t+1} = atx_1 + bt^2 x_{t-1} + sin(t)$ is a

   (i) non-autonomous

   (ii) non-homogeneous

   (iii) linear, provided a and b are constants

   (iv) All of the above

   (Choose the correct answer)

(b) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Write the eigenvalues of the characteristic equation.

(c) Write the condition that a non-negative matrix A is a primitive.

(d) Why does a logistic growth often referred as a sigmoid growth ?

(e) The Juri condition *or* Juri test is satisfied for local asymptotic stability. *(Write true/false)*

(f) Give an example of Malthusian growth.

(g) Write when a system is said to be persistent.

(h) Write a difference between continuous growth and discrete growth.

(i) Lyapunov stability is a technique that can be used to show ________ stability of an equilibrium. *(Fill up the blank)*

(j) Give an example of a simple SI model with no births and deaths.

2. Answer the following questions : 2×5=10

(a) Write *two* basic requirements of a mathematical problem to be properly posed.

(b) Define a dominant eigenvalue.

(c) Check whether the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

satisfies Leslie matrix or not.

(d) Find the equilibria for the difference equation $x_{t+1} = ax_t^3,\ a > 0$.

(e) Write Routh-Hurwitz criteria for the given polynomial

$$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + \dots + a_{n-1}\lambda + a_n,$$

where the coefficients a_i are real constants, $i = 1, 2, \dots, n$.

3. Answer ***any four*** questions : 5×4=20

(a) Find the general solution of the homogeneous difference equation

$$x_{t+2} - 16x_t = 0.$$

(b) Find the eigenvalues and then find the general solution to $X(t+1) = AV(t)$

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

(c) Find all the equilibria for the difference equation

$$x_{t+1} = ax_t^3 + a_t \text{ , } a \neq 0.$$

Then determine whether they are locally asymptotically stable or unstable.

(d) A mathematical model for the growth of a population is

$$\frac{dx}{dt} = \frac{2x^2}{1+x^4} - x = f(x), \; x(0) \geq 0$$

where x is the population density. Find the equilibria and determine their stability.

(e) With a suitable example, explain briefly phase plane analysis.

(f) The normalized form of a predator prey model has the form

$$x_{t+1} = (r+1)x_t - rx_t^2 - Cx_t y_t$$

$$y_{t+1} = Cx_t y_t + (1-d)y_t$$

where r, $c > 0$ and $0 < d < 1$.

Determine conditions for local stability of the equilibria.

4. Answer the following questios : 10×4=40

(a) Discuss a Predator-Prey model with a suitable example by finding its equilibria, local stability. 10

Or

Discuss the behaviour of the system

$$\dot{x} = -ax + gx^2 - bxy$$

$$\dot{y} = -cy + dxy$$

where a, b, c, d and g are all constants.

(b) Suppose the Leslie matrix is given by

$$L = \begin{bmatrix} 0 & \frac{3a^2}{2} & \frac{3a^3}{2} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}, a > 0$$

(i) Find the characteristic equation, eigenvalues and inherent net reproduction number R_0 of L.

(ii) Show that L is primitive

(iii) Find the stable age distribution. 2+3+5=10

Or

Show that the solution the pharmacokinetics model is

$$x(t) = \frac{1}{a}\left(1 - e^{-at}\right)$$

$$y(t) = \frac{1}{t} + \frac{e^{-at}}{a-b} - \frac{ae^{-bt}}{b(a-b)}$$ 10

(c) Discuss the system

$$\dot{x} = ax - gx^2 - bxy$$

$$\dot{y} = c'y - g'y^2 + dxy$$

in the phase plane, all the constants being positive. Will the predators die out if there is no prey? 10

Or

A second order difference equation

$x_{t+2} + ax_{t-1} + bx_t = 0, a \neq 0$ has a characteristic polynomial with a root of multiplicity two, $\lambda_1 = \lambda_2 \neq 0$.

(i) Show that $t\lambda_1^t$ is a solution of the difference equation.

(ii) Show that the casoration $C\left(\lambda_1^t, t\lambda_1^t\right) \neq 0$ for any t. 5+5=10

(d) With a suitable example discuss briefly a SIR epidemic model. 10

Or

Discuss briefly about simple Kermack-McKendric epidemic model.
