2025

PHYSICS

Paper : PHY0400204

(Quantum Mechanics)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

Symbols have their usual meanings.

1. Objective-type : *(Answer **all** questions)* 1×5=5

   (a) If λ_c is the Compton shift, what is the greatest wavelength change in Compton scattering ?

   (b) What is the outcome of $\left[\hat{x}, e^{\hat{x}}\right]$?

(c) Plot the wavefunction $\psi(x) = \frac{1}{a^2} x e^{-x/a}$ for $x > 0$, where a is a constant and real number.

(d) What is the requirement of de Broglie wavelength of electron for the diffraction of electrons by a crystal?

(e) What is the total degeneracy in energy of H-atom with principal quantum number $n = 2$?

2. Very short answer-type : *(Answer **any five** questions)* 2×5=10

(a) Write Planck's blackbody radiation formula and obtain Rayleigh-Jeans formula under limiting condition.

(b) Write the general form of the eigenvalue equations for the Hamiltonian of a one-dimensional linear harmonic oscillator and mention possible eigenvalues.

(c) What is the minimum value of the product $\Delta x \Delta p_x$? Plot Δp_x versus Δx. 1+1=2

(d) Why group velocity and not the phase velocity is considered to describe the velocity of a moving material particle?

(e) What is zero-point energy? Why it cannot be equal to zero for a particle confined in a potential box? 1+1=2

(f) If λ_p and λ_α are the de Broglie wavelengths of a proton and an alpha particle moving with same non-relativistic speeds, then find the ratio $\frac{\lambda_p}{\lambda_\alpha}$.

(g) Write the differential forms of linear momentum and energy operators. Is momentum operator Hermitian? 1+1=2

(h) An attempt is made to measure the position of an electron in an atom. The uncertainty of this measurement is 1Å. What is the minimum uncertainty in the measurement of linear momentum of the electron?

(i) Using the general expression for spherical harmonics $Y_l^{m_l}(\theta, \varphi)$, evaluate Y_1^0.

(j) Write the eigenvalue equations for the z-component of orbital angular momentum and square of the orbital angular momentum operators for a particle under the action of a spherically symmetric potential.

3. Short answer-type : *(Answer **any four** questions)* 5×4=20

(a) Obtain the normalization constant by normalizing the given wavefunction

$$\psi(x) = \frac{3}{\sqrt{10}}(a^2 - x^2) \text{ in the region}$$

$-a \leq x \leq a$. Hence, show the variation of the normalized wavefunction with x graphically with a mention of the peak value. 3+2=5

(b) An experiment on photoelectric effect is conducted for a metal. The stopping potentials are $4.50V$ and $0.20V$ corresponding to light wavelengths $190nm$ and $550nm$, respectively. Find the work function of the metal.

(c) Provide a brief physical interpretation of wavefunction.

(d) Consider a beam of particles of mass m, moving in the positive x direction with energy E towards a potential step at $x = 0$. The potential $V(x)$ is zero for $x \leq 0$ and it is $\frac{3}{4}E$ for $x > 0$. Find the reflection coefficient.

(e) Starting with time dependent Schrödinger equation, obtain the differential form of equation of continuity involving the probability current density.

(f) Briefly describe the Davisson-Germer experiment that confirms wave nature of electrons.

(g) Write the time-independent Schrödinger equation for a one-dimensional linear harmonic oscillator and provide its ground state solution using the Hermite polynomials.

(h) Find the outcome of the commutation relation,

$$\left[\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{y}\hat{p}_z - \hat{z}\hat{p}_y\right]$$

4. Essay-type : *(Answer **any one** question)*

10×1=10

(a) Starting with the concept of wave packet, obtain the intensity distribution. Introduce the Gaussian form of wave packet and briefly explain its connection with a moving material particle. 7+3=10

(b) Write the time independent Schrödinger equation in three dimensions for a particle experiencing central potential and obtain its radial and angular parts in spherical polar coordinate system. Using the idea of separation of variables, find the normalized azimuthal wavefunction. 7+3=10

(c) Consider a particle inside a one-dimensional potential box having infinite potential barriers at $x=0$ and $x=L$. The wavefunction of the particle is $\psi(x)=Nx(L-x)$, where N is the normalization constant. Find the expectation values of position and linear momentum operators.

———