Total number of printed pages–7

3 (Sem–4/CBCS) MAT HC 1

2025

MATHEMATICS

(Honours Core)

Paper : MAT–HC–4016

(Multivariate Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times10=10$

(a) Let $f(x,y)=x^2y+xy^2$, if t is a real number then find $f(1-t,t)$.

(b) Evaluate $\underset{(x,y)\to(0,0)}{Lt}\dfrac{e^x \tan^{-1}y}{y}$

(c) Determine $\dfrac{\partial z}{\partial x}$, if $3x^2+4y^2+2z^2=5$.

(*d*) Define harmonic function.

(*e*) Find $\nabla f(x,y)$ for $f(x,y) = x^2y + y^3$

(*f*) Evaluate $\int_0^4 \int_0^{4-x} xy\,dy\,dx$.

(*g*) Define relative extrema for a function of two variables.

(*h*) Compute $\int_1^4 \int_{-2}^3 \int_2^5 dx\,dy\,dz$.

(*i*) What is the del operator?

(*j*) What is a vector field?

2. Answer the following questions : 2×5=10

(*a*) Determine f_x and f_y for

$$f(x,y) = x^2 e^{x+y} \cos y$$

(*b*) Evaluate $\int_1^2 \int_0^{\pi} x \sin y\,dy\,dx$

(*c*) Find the Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$ when

$$u = x - 2y,\ v = 3x - 5y.$$

(*d*) Find the curl of the vector field

$$\bar{F} = x^2yz\,\hat{i} + xy^2z\,\hat{j} + xyz^2\,\hat{k}.$$

(*e*) Explain the difference between $\int_C f\,ds$ and $\int_C f\,dx$.

3. Answer ***any four*** questions : 5×4=20

(*a*) Compute the slope of the tangent line to the graph of $f(x,y) = x^2 \sin(x+y)$ at the point $P_0\left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$.

(*b*) Find all relative extrema and saddle points of the function

$$f(x,y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(c) Evaluate

$$\iint_R x^2 e^{xy} dA;\ R: 0 \le x \le 1,\ 0 \le y \le 1\ .$$

(d) Evaluate $\iiint_D x\,dV$, where D is the solid in the first octant bounded by the cylinder $x^2 + y^2 = 4$ and the plane $2y + z = 4$.

(e) Find the volume of the solid in the first octant that is bounded by $x^2 + y^2 = 2y$, the half–cone $z = \sqrt{x^2 + y^2}$, and the xy–plane.

(f) If $\vec{F}(x,y,z) = xy\,\hat{i} + yz\,\hat{j} + z^2\,\hat{k}$ and $\vec{G}(x,y,z) = x\hat{i} + y\,\hat{j} - z\hat{k}$ then find curl $(F \times G)$.

4. Answer ***any four*** questions : 10×4=40

(a) Let $f(x,y) = \begin{cases} xy\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right), & (x,y) \ne (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$

Show that $f_x(0,y) = -y$ and $f_x(x,0) = x$ for all x and y. Then show that $f_{xy}(0,0) = -1$ and $f_{yx}(0,0) = 1$.

(b) When two resistors with resistances P and Q ohms are connected in parallel, the combine resistance is R, where

$$\frac{1}{R} = \frac{1}{P} + \frac{1}{Q}$$

If P and Q are measured at 6 and 10 ohms respectively, with error no greater than 1%, what is the maximum percentage error in the computation of R?

(c) (i) If f is differentiable and $z = u + f\left(u^2 v^2\right)$. Show that

$$u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = u\ .$$ 5

(ii) If $f(x,y)$ is a homogeneous function of degree n, show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf\ .$$ 5

(d) (i) Define directional derivative. 2

(ii) Let $f(x,y,z) = xyz$, and let $\hat{u}$ be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1,-1,2)$ in the direction of $\vec{u}$. 8

(e) (i) Find $div\,\vec{F}$, given that $\vec{F} = \nabla f$, where $f(x,y,z) = xy^3z^2$. 4

(ii) If $\vec{F}(x,y) = u(x,y)\hat{i} + v(x,y)\hat{j}$, Show that

Curl $\vec{F} = 0$ if and only if $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$. 6

(f) Let $\vec{F} = xy^2\,\hat{i} + x^2y\,\hat{j}$ and evaluate the line integral $\int_C \vec{F}.d\vec{R}$ between the points $(0,0)$ and $(2,4)$ along the following path :

(i) the line segment connecting the points. 4

(ii) the parabolic are $y = x^2$ connecting the points. 6

(g) Evaluate the line integral

$$\oint_C \frac{x\,dy - y\,dx}{x^2 + y^2}$$

where C is the unit circle $x^2 + y^2 = 1$ traversed once counter clockwise.

(h) Show that the vector field $\vec{F} = \left(e^x \sin y - y\right)\hat{i} + \left(e^x \cos y - x - 2\right)\hat{j}$ is conservative and then find a scalar potential function f for $\vec{F}$.
