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3 (Sem-2/CBCS) MAT HC 1

2025

MATHEMATICS

(Honours Core)

Paper : MAT-HC-2016

(Real Analysis)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :
1×10=10

- (a) If $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$, then find the value of $\inf S$.
- (b) "The unit interval $[0,1]$ is uncountable."
(State True or False)
- (c) State Bolzano's Intermediate Value theorem.

- (d) Give an example of a bounded sequence that is not a Cauchy sequence.
- (e) "Every bounded sequence with a unique limit point is convergent."
(State True or False)
- (f) Give an example of a set which is not bounded above.
- (g) For what value of ℓ , $\lim_{n \rightarrow \infty} \left(n \log \frac{u_n}{u_{n+1}} \right) = \ell$ converges where $\sum u_n$ is a positive term series.
- (h) Define an alternating series.
- (i) "Every absolutely convergent series is convergent."
(State True or False)
- (j) What is the value of $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$.

2. Answer the following questions: $2 \times 5 = 10$

- (a) If $S = \{1 - (-1)^n / n : n \in N\}$, find $\inf S$.
- (b) Prove that $\lim (1 / (n^2 + 1)) = 0$.

- (c) Examine the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \dots\right)$ is convergent or divergent.
- (d) Define Cauchy sequence.
- (e) If $a, b \in R$, then show that $a^2 + b^2 = 0$ if and only if $a = 0$ and $b = 0$.

3. Answer **any four** questions: $5 \times 4 = 20$

- (a) Show that the set S of real number is bounded iff there exists a real number $G > 0$ such that $|x| \leq G, \forall x \in S$.
- (b) Prove that if $I_n = [a_n, b_n], n \in N$ is a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $\inf \{b_n - a_n : n \in N\} = 0$, then the number ξ contained in I_n for all $n \in N$.
- (c) Prove that a sequence in R can have at most one limit.
- (d) Prove that if $X = (x_n)$ is a convergent sequence of real numbers and if $x_n \geq 0$, for all $n \in N$, then $x = \lim(x_n) \geq 0$.

(e) Examine the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is not convergent.

(f) Prove that if $\sum u_n$ is a positive term series, such that $\lim_{n \rightarrow \infty} (u_n)^{\frac{1}{n}} = \ell$, then the series

(i) converges, if $\ell < 1$;

(ii) diverges, if $\ell > 1$.

4. Answer the following questions: $10 \times 4 = 40$

(a) State and Prove Bolzano-Weierstrass Theorem.

Or

Test for convergence of the series whose

n th term is $\left\{ (n^2 + 1)^{\frac{1}{3}} - n \right\}$.

(b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $0 < p \leq 1$.

Or

(i) Show that the series

$$\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \dots + \left(\frac{1}{(\log n)^p} \right)$$

diverges for $p > 0$.

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(ii) For real number $x, a, \varepsilon > 0$, show that

$$(i) \quad |x| < \varepsilon \Leftrightarrow -\varepsilon < x < \varepsilon$$

$$(ii) \quad |x - a| < \varepsilon \Leftrightarrow a - \varepsilon < x < a + \varepsilon$$

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(c) If $\{a_n\}$ and $\{b_n\}$ are bounded sequences, then

$$(i) \quad \underline{\lim} a_n + \underline{\lim} b_n \leq \underline{\lim} (a_n + b_n)$$

$$(ii) \quad \underline{\lim} a_n + \overline{\lim} b_n \leq \overline{\lim} (a_n + b_n)$$

Or

- (i) Test the convergence of the

series $\sum \frac{1}{n^{1+1/n}}$ 5

- (ii) Test the convergence of the series

$\frac{\alpha}{\beta} + \frac{(1+\alpha)}{(1+\beta)} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \dots$ 5

- (d) (i)

$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots$

5

- (ii) Show that the sequence

$a_{n+1} = \frac{1}{2} \left(a_n + \frac{9}{a_n} \right), n \geq 1 \text{ and } a_1 > 0$

converges to 3. 5

Or

- (i) Show that for any fixed value of

x the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is

convergent. 5

- (ii) Show that

$\lim_{n \rightarrow \infty} \frac{m(m-1)\dots(m-n+1)}{(n-1)!} x^n = 0,$

where $|x| < 1$ and m is any real number. 5