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3 (Sem–6/CBCS) PHY HE 3

2025

PHYSICS

(Honours Elective)

Paper : PHY–HE–6036

(Advanced Mathematical Physics–II)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 10 = 10$

   (a) A system of five particles has seven holonomic constraints. Mention the number of generalized co-ordinates to describe the system.

(b) Give reason why the following function cannot be a probability function.

$$f(x)=\begin{cases}\frac{1}{6} & \text{for } x=-5\\ \frac{-5}{6} & \text{for } x=1\\ 0 & \text{elsewhere}\end{cases}$$

(c) Give example of a cyclic group.

(d) Express Lagrange Bracket in terms of Jacobian.

(e) Suppose that X has a Poisson distribution with parameter λ. Then

(i) $E(X)=Var(X)$

(ii) $E(X)<Var(X)$

(iii) $E(X)>Var(X)$

(iv) $E(X)$ and $Var(X)$ are not linked at all

(f) Write the Hamiltonian of a projectile in space.

(g) Is the following statement correct?

"The set $Z(G)=\{Z\in G\forall g\in G, Zg\neq gZ\}$ is the centre of a group.

(h) Mention why $(G,\circ)$ is not a subgroup of $(R,+)$ where the numbers in group G and R are all positive real numbers.

(i) Fill in the blank:

In a Poisson distribution,

$2P(x=1)=P(x=2)$ the standard deviation is ______.

Or

R is a transitive relation in a set A then

aRb and ____ R ____ $\Rightarrow$ ____ RC.

(j) The number of elements in a permutation on n elements is

(i) n

(ii) $\lfloor n$

(iii) ${}^nC_n$

(iv) ${}^nP_0$

2. Answer the following questions: 2×5=10

(a) If $A=\{3,5\}$ and $B=\{p,q,r,s,t\}$ then find the total number of relations from A to B.

(b) Let a mapping be defined by $f:(I,+)\to f:(I_e,+),\ f(n)=2n\ \forall n\in I$, check whether f is a homomorphism of I Into I_e.

(c) If A and B are two events and if

$$P(A)=\frac{1}{2},\ P(B)=\frac{1}{4} \text{ and } P(A\cup B)=7/8$$

then find $P(A\mid\bar{B})$.

(d) What are generalised co-ordinates? Explain with examples.

Or

What are holonomic and non-holonomic constraints? Give examples.

(e) Mention *at least four* characteristics of Normal distribution.

3. Answer ***any four*** of the following : 5×4=20

(a) Define cosets of a group G. Show that for an abelian group there is no distinction between left and right cosets. 1+4=5

(b) Define a conservative system. Establish Euler-Lagrange's equations of motion in a conservative system. 1+4=5

(c) Use Lagrange's equations to find the equation of motion of a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

(d) Do the integers with respect to multiplicative binary operation form a group? Explain. 1+4=5

(e) Show that if α, β are two constants of motion then their Poisson bracket is likewise a constant of motion.

(f) Explain what do you mean by permutation in group theory.

Show that if the permutation

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix}$$ be multiplied three

times to itself it will give an identity permutation. 2+3=5

4. Answer ***any four*** of the following: 10×4=40

(a) (i) Find the extremal of the function

$$\int_0^1\left[y'^2+12xy\right]dx \text{ with } y(0)=0,$$

$y(1)=1$. 5

(ii) Define an abelian group.

If $a^2 = e$ for any $a \in G$, then show that G is abelian, e being the identity element. 1+4=5

(b) (i) Explain Legendre transformation for function of two variables in Classical Mechanics. 4

(ii) Define canonical pair of variables in Classical Hamiltonian Mechanics. 1

(iii) The Lagrangian of a particle of mass m moving in a plane is given

by $L = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2\right) + a\left(x\dot{y} + y\dot{x}\right)$.

Find out canonical momenta and the Hamiltonian.

Also write down the Lagrangian of a charged particle in an electromagnetic field. 2+2+1=5

(c) (i) Define conditional probability.

Two cards are drawn at rando and without replacement from a pack of 52 playing cards. Find the probability that both the cards are red. 1+3=4

(ii) State and prove Bayes' theorem. 2+4=6

(d) (i) Evaluate mean value and variance of x for the radius of an electrical fuse wire following the probability distribution function

$$f(x) = \frac{3}{4}\left(2 - x^2\right),\ 0 \le x \le 2. \qquad 7$$

(ii) If the random variable X has a Poisson distribution such that

$P(X = 3) = P(X = 4)$, find

$P(X = 5)$. Given $e^{-4} = 0{\cdot}0183$. 3

(e) (i) Define Hamiltonian, H of a system. What is its dimension? E is the total energy of a system, establish the conditions of equality of H and E. 1+1+5=7

(ii) Explain Hamilton's variational principle. 3

(f) (i) Mention *one* merit and *one* demerit of least square method.

Fit a least square parabola $Y = AX^2 + BX + C$ to the data in the adjoining table—

X	:	0	1	2	3	4
Y	:	2·4	2·1	3·2	5·6	9·3

1+1+6=8

(ii) Write the equation of the least square line corresponding to the relation for focal length of a lens. 2

(g) Define a random variable. Let X denotes the sum of numbers on two fair dice. What is the mathematical expectation of X. Obtain $E\left(X^2\right)$. Check wether

$E\left(X^2\right) = \left[E(X)\right]^2$. 1+5+3+1=10

(h) A particle moves on a smooth curve, joining the two fixed points A and B, under gravity, starting from rest at A. Find the form of the path in order that the time from A to B is minimum. 10

Or

Explain variational principle in physics. Find out the equation of Geodesic on sphere. 4+6=10
