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**3 (Sem - 1 / CBCS) PHY HC 1**

**2021**

**( Held in 2022 )**

**PHYSICS**

(Honours)

Paper : PHY-HC-1016

**( Mathematical Physics - I )**

Full Marks : 60

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions :  $1 \times 7 = 7$ 
  - (a) State the vector field with respect to Cartesian co-ordinate. Give *one* example.
  - (b) Show that  $\vec{\nabla} \cdot \vec{r} = 3$ , where  $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ .

Contd.

- (c) Write the order and degree of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

- (d) Write the volume element in curvilinear co-ordinate.

(e) Give the value of  $\int_{-\alpha}^{+\alpha} \delta(x) dx$

- (f) Define variance in statistics.

- (g) State the principle of least square fit.

2. Answer the following questions :

$$2 \times 4 = 8$$

- (a) Find a unit vector perpendicular to the surface,  $x^2 + y^2 - z^2 = 11$  at the point (4, 2, 3).

- (b) If  $\vec{A} = \vec{A}(t)$ , then show that

$$\frac{d}{dt} \left[ \vec{A} \cdot \left( \frac{d\vec{A}}{dt} \times \frac{d^2\vec{A}}{dt^2} \right) \right] = A \cdot \left[ \frac{d\vec{A}}{dt} \times \frac{d^3\vec{A}}{dt^3} \right]$$

- (c) If  $\vec{A}$  and  $\vec{B}$  are each irrotational, prove that  $\vec{A} \times \vec{B}$  is solenoidal.

- (d) Evaluate  $\iint_S \vec{r} \times \hat{n} dS$ , where S is a closed surface.

3. Answer **any three** of the following questions :

$$5 \times 3 = 15$$

- (a) Prove

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \iint_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) \cdot d\vec{S}$$

- (b) Find the integrating factor (IF) of the following differential equation and

solve it.

$$(1+x^2) \frac{dy}{dx} + 2xy = \cos x$$

(c) Express curl  $\vec{A} = \vec{\nabla} \times \vec{A}$  in cylindrical co-ordinate.

(d) What is Dirac-delta function ? Show that the function

$$\delta(x) = \lim_{\epsilon \rightarrow 0} \frac{\sin(2\pi\epsilon x)}{\pi\epsilon}$$

is a Dirac delta function.

(e) If  $\phi(x, y, z) = 3x^2y - y^3x^2$  be any scalar function  $\phi$ , find out

(i) grad  $\phi$  at point (1, 2, 2)

(ii) unit vector  $\hat{e}$  perpendicular to surface.

4. Answer **any three** of the following questions : 10×3=30

(a) (i) If  $F_1(x, y), F_2(x, y)$  are two continuous functions having continuous partial derivatives

$\frac{\partial F_1}{\partial y}$  and  $\frac{\partial F_2}{\partial x}$  over a region  $R$

bounded by simple closed curve  $C$  in the  $x$ - $y$  plane, then show that

$$\oint_C (F_1 dx + F_2 dy) = \iint_S \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx dy \quad 7$$

(ii) A function  $f(x)$  is defined

$$\text{as } \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \leq x \leq 4 \\ 0, & x > 4 \end{cases}$$

Show that it is a probability density function. 3

(b) Solve the following differential equations : 5+5=10

(i)  $9 \frac{d^2y}{dx^2} + 12 \frac{dy}{dx} + 4y = 6e^{-2x/3}$

(ii)  $2 \frac{\partial^2 z}{\partial x^2} + 5 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = 0$

(c) (i) A rigid body rotates about an axis passing through the origin with angular velocity  $\vec{\omega}$  and with linear velocity  $\vec{v} = \vec{\omega} \times \vec{r}$ , then prove that,

$$\vec{\omega} = \frac{1}{2} (\vec{\nabla} \times \vec{v})$$

where,  $\vec{\omega} = \hat{i}\omega_1 + \hat{j}\omega_2 + \hat{k}\omega_3$

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z \quad 5$$

(ii) If  $y = f(x+at) + g(x-at)$ , show that it satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

where  $f$  and  $g$  are assumed to be at least twice differentiable and  $a$  is any constant. 5

(d) (i) Apply Green's theorem in plane to evaluate the integral

$$\oint_C [(xy - x^2) dx + x^2 y dy] \text{ over the triangle bounded by the line } y=0, x=1 \text{ and } y=x. \quad 6$$

(ii) Prove that

$$\int_{-\alpha}^{+\alpha} f(x) \delta(x-c) dx = f(c) \quad 4$$

(e) (i) Applying Gauss' theorem, evaluate

$$\iiint_S x dy dz + y dz dx + z dx dy, \text{ where}$$

$S$  is the sphere of radius  $x^2 + y^2 + z^2 = 1$  5

(ii) Evaluate  $\nabla^2 \psi$  in spherical co-ordinate. 5