

Total number of printed pages-7

3 (Sem-3/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY-HC-3016

(Mathematical Physics-II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$
 - (a) Define the ordinary point of a second order differential equation.
 - (b) Show that $P_0(x) = 1$.
 - (c) Write the Laplace equation in spherical polar co-ordinate system.
 - (d) A partial differential equation has
 - (i) one independent variable

Contd.

- (ii) more than one dependent variable
- (iii) two or more independent variables
- (iv) no independent variable.

(Choose the correct option)

- (e) If A and B are two square matrices of order n , show that

$$\text{Trace}(A + B) = \text{Trace } A + \text{Trace } B$$

- (f) Which one of the following is the value of $\Gamma\left(\frac{1}{2}\right)$?

(i) $\sqrt{\frac{\pi}{2}}$

(ii) $\sqrt{\pi}$

(iii) π

(iv) $\sqrt{\pi}/2$

- (g) Define self adjoint of a matrix.

2. Answer the following questions: $2 \times 4 = 8$

- (a) Check the behaviour of point $x = 0$ for the differential equation

$$\frac{d^2 y}{dx^2} - \frac{6}{x} y = 0$$

- (b) If $\int_{-1}^{+1} P_n(x) dx = 2$, find the value of n .

- (c) Express the Fourier series in complex form.

- (d) Verify the matrix

$$A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix} \text{ is a unitary}$$

matrix.

3. Answer **any three** of the following questions: $5 \times 3 = 15$

- (a) Find the power series solution of the following differential equation:

$$(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

about $x = 0$.

- (b) Define Gamma function. Show that

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \quad 1 + 4 = 5$$

(c) Establish the following recurrence formula for Legendre polynomial $P_n(x)$

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

(d) Show that the Fourier expansion of the function $f(x) = x$, $0 < x < 2\pi$ is

$$x = \pi - 2 \left[\sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x + \dots \right]$$

(e) What is eigenvalue of a matrix? Show that the eigenvalues of Hermitian matrix are real. 1+4=5

4. Answer **any three** of the following questions : 10×3=30

(a) (i) If $P_n(x)$ be the polynomial of Legendre's differential equation, show that

$$P_n(x) = \frac{1}{2^n \cdot n!} \cdot \frac{d^n}{dx^n} (x^2 - 1)^n \quad 6$$

(ii) Prove that 4

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$$

(b) (i) What is Beta function? Show that

$$(a) \beta(1, 1) = 1$$

$$(b) \beta(m, n) = \beta(n, m) \quad 1+1+3=5$$

(ii) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that

$A^2 - 4A - 5I = 0$ where I and O are the unit matrix and the null matrix of order 3 respectively. Also use this result to find A^{-1} . 3+2=5

(c) Find the Fourier series expansion of

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Also show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad 6+4=10$$

(d) (i) Diagonalize the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad 5$$

- (ii) Show that the generating function for Hermite polynomial $H_n(x)$, for integral n and real value of n is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x) \quad 5$$

- (e) (i) Write the one dimensional diffusion equation (heat flow equation) and find the general solution of the same by the method of separation of variable.

1+7=8

- (ii) For the Pauli spin matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \text{ and}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ show that}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3 \quad 2$$

- (f) (i) Write the Orthogonality conditions of sine and cosine functions.

$$1\frac{1}{2} + 1\frac{1}{2} = 3$$

- (ii) A square wave function is represented as

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x < 0 \\ h, & \text{for } 0 \leq x < \pi \end{cases}$$

Draw the graphical representation of the wave function and expand the same in Fourier series.

1+6=7